WISKUNDIGE LOGICA (2016-2017)
HOMEWORK 6

- Deadline: May 19 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Success!

(1) (30pt)

(a) Show that if $\sim_F$ is an equivalence relation on $\prod_{I\in I} \mathfrak{A}_i$, then F is a filter on I, provided that each $\mathfrak{A}_i$ contains more than two elements. (This is the converse of Lemma 1.1 of Chapter 5 of Bell and Slomson).

(b) Let F be the filter $\{I\}$ on I. Prove that

$$\prod_{I\in I} \mathfrak{A}_i/F \cong (\prod_{I\in I} \mathfrak{A}_i, S)$$

(c) Construct a counterexample to (a) in the case where some of the $\mathfrak{A}_i$, contain only two elements.

(2) (20pt) Give a definition, either in Φ_0 (see VII.§2) or ZFC (see VII.§3), for the following two statements:

- $x = y \times z$
- $x = \bigcup y$

(3) (20pt) Let (x, x') and (y, y') be ordered pairs. Show, either in Φ_0 (see VII.§2) or ZFC (see VII.§3), that

$$(x, x') = (y, y') \text{ iff } x = x' \text{ and } y = y'.$$

(4) (30pt) Show that $\{\emptyset, \{\emptyset\}\}$ is a set in ZFC. (For the axioms of ZFC see VII.§3.)

(5) (**Bonus** (10pt)) Show that there exists an uncountable set in ZFC.