14 Uitwerkingen extra opgaven: limieten

Opgave 14.1.

a.

$$\begin{aligned}\lim_{n\to\infty} \frac{\cos(1/n)}{n} &= \lim_{n\to\infty} \frac{1}{n} \cdot \cos(1/n) = \left(\lim_{n\to\infty} \frac{1}{n}\right) \cdot \left(\lim_{n\to\infty} \cos(1/n)\right) \\ &= \left(\lim_{n\to\infty} \frac{1}{n}\right) \cos(0) = \lim_{n\to\infty} \frac{1}{n} = 0\end{aligned}$$

b.

$$\begin{aligned}\lim_{n\to\infty} \frac{e^{\sqrt{1+\frac{1}{n}}} - 1}{\ln(e + 2^{-n})} &= \frac{e^{\sqrt{1+\lim_{n\to\infty} 1/n}} - 1}{\ln(e + \lim_{n\to\infty} 2^{-n})} = \frac{e^{\sqrt{1+0}} - 1}{\ln(e+0)} \\ &= \frac{e^{\sqrt{1}} - 1}{\ln(e)} = \frac{e-1}{1} = e - 1\end{aligned}$$

c.

$$\begin{aligned}\lim_{n\to\infty} \frac{1 + 2^{1/n}}{\sqrt{2 + 1/n^2}} &= \frac{1 + \lim\limits_{n\to\infty} 2^{1/n}}{\sqrt{2 + \lim\limits_{n\to\infty} 1/n^2}} = \frac{1 + 2^0}{\sqrt{2+0}} \\ &= \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\end{aligned}$$

Opgave 14.2. Bij deze opgave gebruiken we de bijzondere limieten

a. $\lim\limits_{n\to\infty} 2n \sin(1/n) = 2 \cdot \lim\limits_{n\to\infty} n \sin(1/n) = 2 \cdot 1 = 2$

b. $\lim\limits_{n\to\infty} \left(1 - \frac{1}{n}\right)^n = \lim\limits_{n\to\infty} \left(1 + \frac{-1}{n}\right)^n = e^{-1} = \frac{1}{e}$

c. $\lim\limits_{n\to\infty} \left(1 + \frac{1}{2n}\right)^n = \lim\limits_{n\to\infty} \left(1 + \frac{1/2}{n}\right)^n = e^{1/2} = \sqrt{e}$

Opgave 14.3.

a.

$$\begin{aligned}\lim_{n\to\infty} \frac{n^3}{2-n^2} \sin(1/n) &= \lim_{n\to\infty} \frac{n^2}{2-n^2} \cdot n \sin(1/n) = \lim_{n\to\infty} \left(\frac{n^2}{2-n^2}\right) \cdot \lim_{n\to\infty} \left(n \sin\left(\frac{1}{n}\right)\right) \\ &= \lim_{n\to\infty} \left(\frac{1}{\frac{2}{n^2} - 1}\right) \cdot 1 = \frac{1}{0-1} = -1\end{aligned}$$

b.

$$\begin{aligned}\lim_{n\to\infty} \frac{2^n + 1}{2^n} \sqrt{\left(1 + \frac{4}{n}\right)^n} &= \lim_{n\to\infty} \frac{2^n+1}{2^n} \cdot \lim_{n\to\infty} \sqrt{\left(1 + \frac{4}{n}\right)^n} \\ &= \lim_{n\to\infty} \frac{1 + 2^{-n}}{1} \cdot \sqrt{\lim_{n\to\infty} \left(1 + \frac{4}{n}\right)^n} \\ &= \frac{1+0}{1} \cdot \sqrt{e^4} = 1 \cdot \left(e^4\right)^{1/2} \\ &= e^{4 \cdot \frac{1}{2}} = e^2\end{aligned}$$

c.

$$
\begin{aligned}
\lim_{n\to\infty} \frac{n^2}{1+2n} \ln\left(1-\frac{2}{n}\right) &= \lim_{n\to\infty} \frac{n}{1+2n} \cdot n \ln\left(1-\frac{2}{n}\right) \\
&= \lim_{n\to\infty} \frac{n}{1+2n} \cdot \ln\left(\left(1-\frac{2}{n}\right)^n\right) \\
&= \lim_{n\to\infty} \frac{n}{1+2n} \cdot \lim_{n\to\infty} \ln\left(\left(1-\frac{2}{n}\right)^n\right) \\
&= \lim_{n\to\infty} \frac{1}{\frac{1}{n}+2} \cdot \ln\left(\lim_{n\to\infty}\left(1+\frac{-2}{n}\right)^n\right) \\
&= \frac{1}{0+2} \cdot \ln\left(e^{-2}\right) \quad = \quad \frac{1}{2} \cdot -2\ln(e) \\
&= -1 \cdot 1 \quad = \quad -1
\end{aligned}
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