

RULES OF SEQUENT CALCULUS

$$\text{(Ant)} \quad \frac{\Gamma \vdash \varphi}{\Gamma' \vdash \varphi} \quad \text{if } \Gamma \subseteq \Gamma'$$

$$\text{(Asm)} \quad \frac{}{\Gamma \vdash \varphi} \quad \text{if } \varphi \in \Gamma$$

$$\text{(PC)} \quad \frac{\Gamma, \psi \vdash \varphi \quad \Gamma, \neg\psi \vdash \varphi}{\Gamma \vdash \varphi}$$

$$\text{(Ctr)} \quad \frac{\Gamma, \neg\varphi \vdash \psi \quad \Gamma, \neg\varphi \vdash \neg\psi}{\Gamma \vdash \varphi}$$

$$\text{(\forall A)} \quad \frac{\Gamma, \varphi \vdash \chi \quad \Gamma, \psi \vdash \chi}{\Gamma, (\varphi \vee \psi) \vdash \chi}$$

$$\text{(\forall S)} \quad \frac{\Gamma \vdash \varphi}{\Gamma \vdash (\varphi \vee \psi)} \quad \frac{\Gamma \vdash \varphi}{\Gamma \vdash (\psi \vee \varphi)}$$

$$\text{(\exists A)} \quad \frac{\Gamma, \varphi_x^y \vdash \psi}{\Gamma, \exists x\varphi \vdash \psi} \quad \text{if } y \text{ is not free in } \Gamma \exists x\varphi \psi$$

$$\text{(\exists S)} \quad \frac{\Gamma \vdash \varphi_x^t}{\Gamma \vdash \exists x\varphi}$$

$$\text{(\equiv)} \quad \frac{}{t \equiv t}$$

$$\text{(Sub)} \quad \frac{\Gamma \vdash \varphi_x^t}{\Gamma, t \equiv t' \vdash \varphi_x^{t'}}$$

$$\text{(Cp)(a)} \quad \frac{\Gamma, \varphi \vdash \psi}{\Gamma, \neg\psi \vdash \neg\varphi}$$

$$\text{(Cp)(b)} \quad \frac{\Gamma, \neg\varphi \vdash \neg\psi}{\Gamma, \psi \vdash \varphi}$$

$$\text{(Cp)(c)} \quad \frac{\Gamma, \neg\varphi \vdash \psi}{\Gamma, \neg\psi \vdash \varphi}$$

$$\text{(Cp)(d)} \quad \frac{\Gamma, \varphi \vdash \neg\psi}{\Gamma, \psi \vdash \neg\varphi}$$

$$\text{(Ctr')} \quad \frac{\Gamma \vdash \psi \quad \Gamma \vdash \neg\psi}{\Gamma \vdash \varphi}$$

$$\text{(Ch)} \quad \frac{\Gamma \vdash \varphi \quad \Gamma, \varphi \vdash \psi}{\Gamma \vdash \psi}$$