WISKUNDIGE LOGICA (2016-2017)
HOMEWORK 4

- Deadline: April 24 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Success!

(1) (30pt)

Let $S = \{R\}$ with unary R and let $\Phi = \{\exists x Rx\} \cup \{\neg Ry \mid y \text{ is a variable}\}$. Show:

(a) Φ is satisfiable and therefore consistent.

(b) For no term $t \in T^S$, $\Phi \vdash Rt$.

(c) If $\mathfrak{I} = (\mathfrak{A}, \beta)$ is a model of Φ, then $A \setminus \{\mathfrak{I}(t) \mid t \in T^S\}$ is nonempty.

(2) (20pt)

Let $S = \{R\}$ with unary R and let x and y be distinct variables. For $\Phi = \{Rx \vee Ry\}$ show

(a) $\Phi \nvdash Rx$ and $\Phi \nvdash \neg Rx$.

(b) $\mathfrak{I}^\Phi \not\models \Phi$.

(3) (10pt) Let $\Phi = \{Rwt, Rzt, Rwt \wedge Rzt \to Rxt, y \equiv t\}$. Does $\mathfrak{I}^\Phi \models Rxy$ hold? Justify your solution.

(4) (40pt) Let S be a symbol set.

(a) Prove: If φ is an S-sentence such that all infinite structures are a model of φ, then there exists a natural number n such that all structures with n or more elements are models of φ.

(b) Refute: If Σ is a set of S-sentences such that all infinite structures are a model of Σ, then there exists a natural number n such that all structures with n or more elements are models of Σ.